Iterative Solution of Linear Systems in Harmonic Balance Analysis.

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Abstract

Harmonic balance (HB) is a steady-state simulation technique of primary interest for RF and microwave circuits. Krylov subspace methods promise efficient solution of the large linear systems that arise in HB simulators. This paper deals with an experimental investigation of GMRES and QMR, two leading Krylov subspace methods as applied to the HB problem. The problem of coordinating the linear solver's accuracy with the error at the nonlinear level is also discussed.

Introduction

Recent advances in techniques of steady-state simulation of RF circuits are based on the use of iterative methods for solving large linear systems[1]. In particular, harmonic balance (HB) [2-4] as a method of numerical steady-state analysis in frequency domain is well suited for exploitation of iterative techniques. The dimension of solved HB problems grows rapidly with increasing circuit size and number of tones and harmonics. Increasing the permitted dimension by the use of iterative methods at the linear level allows simulation of circuits with stronger nonlinearities and extends the opportunity for multitone circuits analysis. As a result such computational tools [5-6] have the potential for a wider application in comparison with traditional tools based on direct methods.

Krylov-subspace iterative methods with preconditioning are the most promising approach for new HB simulators due to their good convergence properties. This class of methods has been studied extensively [10]. The present paper focuses on two computational procedures: the GMRES-method [11], based on an Arnoldi process, and the QMR-method [12], based on a Lanczos process. The new RF simulation tools are primarily based on these methods [6-9], and each of the methods has its own numerical advantages and disadvantages.

Some results of comparison of the GMRES and QMR algorithms in a HB simulator are presented here. We also concentrate on the problems of the algorithmic connection between the nonlinear and linear solvers. We show that it is profitable to set the error tolerance for the linear solver depending on residual norm of nonlinear equations.

Iterative linear solvers

Krylov subspace techniques are well suited to solve large linearized HB problems,

$$Ax = b \qquad (1)$$

where A is a $N \times N$ matrix, $N = N_{eq} \times (2 \times N_f - 1)$, N_{eq} is the number of circuit equations, and N_f is the number of frequencies. The following advantages of Krylov subspace methods for large systems are noteworthy:

- numerical stability due to using orthogonalization techniques;
- they can be matrix implicit, i.e. the matrix A need not be formed explicitly;
- accuracy control during processing.

Krylov subspace algorithms consist of two stages:

- the construction of suitable basis vectors for the Krylov subspaces;
- the choice of the iterates x_l.

Two alternative computational scheme, GMRES and QMR, have been successfully exploited in RF simulation [5-9]. The GMRES algorithm uses an Arnoldi process to construct an orthonormal basis [10,11]. The principle of the Arnoldi approach is to construct an orthonormal basis

$$V_l = (v_1, v_2, \ldots, v_l)$$

and sequential computation of vector v_{j+1} under condition

$$v_{j+1} \perp (v_j, v_{j-1}, \ldots, v_1)$$

As a result $N \times l$ matrix V_l with l Arnoldi vectors and $(l+1) \times l$ Hessenberg form H_l are constructed. The large storage required for these matrices is the main limitation of the GMRES-technique.

An alternative approach to constructing a suitable basis of vectors for the Krylov subspace is the nonsymmetrical Lanczos process [10]. This process can be characterized by short recurrences and significantly lower storage requirements in comparison with the Arnoldi process. Even though each iteration requires two matrix-vector multiplies, when the number of iterations is large, the nonsymmetric Lanczos process is more economical due to the reduction of A to tridiagonal form. New basis vectors v_{j+1} are obtained from orthogonality conditions only to nearest vectors, $v_{j+1} \perp (v_j, v_{j-1})$. The following well known drawbacks have been avoided in the nonsymmetric Lanczos process in recent developments [10]:

- the need of adjoint matrix-vector multiplications;
- the possibility of breakdowns or near breakdowns;
- its irregular convergence.

In particular, the developed QMR technique [12] is directed to solve nonsymmetrical problems and avoids the above mentioned drawbacks. QMR was successfully exploited in circuit simulation and in particular in HB simulators [5,6].

Results

Both the QMR and the GMRES algorithms can achieve higher simulation speed compared to direct methods. The choice of tools with fast algorithms to solve linear problems plays a significant role for HB simulators due to high computational complexity of this intermediate stage in computing the solution of nonlinear problem. We performed a series of experiments to compare the efficiency of the two methods for HB analysis.

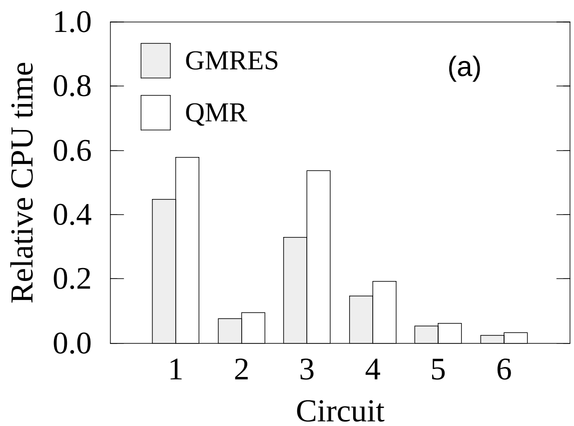
Table 1 contains a brief description of the test problems. We used twenty harmonics for circuits 1 - 5, and 15 harmonics for circuit 6. Two main dependencies were investigated with numerical experiments on these test problems:

Table 1: Test Circuits

Circuit	# nodes	dimension
1. Power supply	6	246
2. Amplifier	8	328
3. Class C amplifier	12	492
4. Differential pair	12	492
5. Opamp	29	1189
6. Five pole active filter	139	4310

CPU time dependence on input signal amplitude and CPU time dependence on the error tolerance specified for the linear solver. The computations were performed by both GMRES and QMR algorithms and also by direct method of Gaussian elimination to estimate acceleration of iterative techniques for these examples. The results are also shown in Fig.1 for small amplitude (Fig. 1a) as well as for large amplitudes (Fig. 1b).

These results demonstrate the advantages of GMRES techniques. Moreover the time spent versus growth of accuracy (1/eps) were obtained



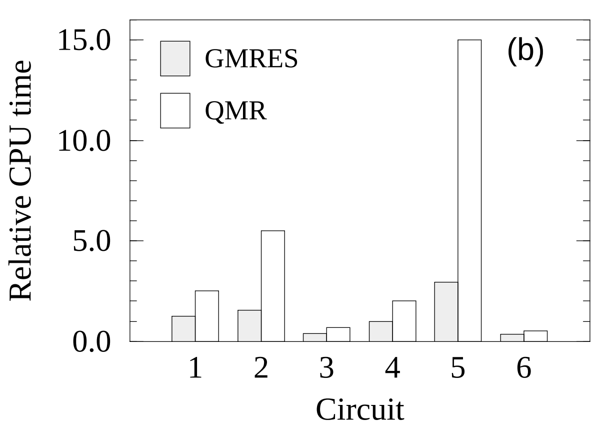
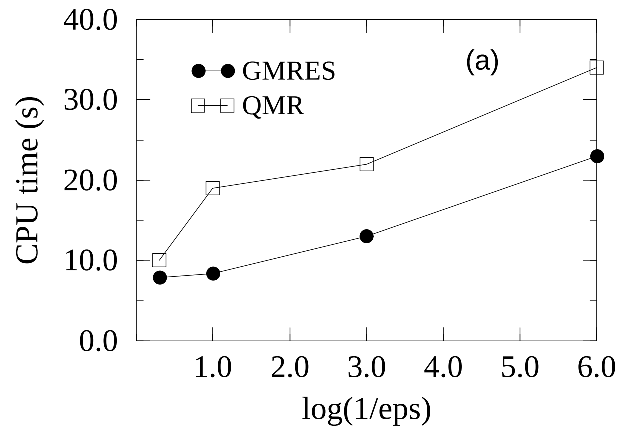


Figure 1. Comparison of GMRES and QMR CPU time relative to direct Gaussian elimination for (a) small and (b) large amplitude input and eps = 1e-6.



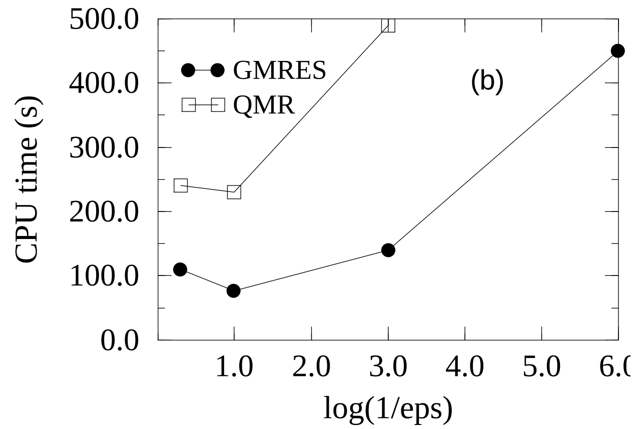


Figure 2. Comparison of GMRES and QMR CPU time for circuit 5 driven with a (a) small amplitude and (b) large amplitude input.

for the circuits. The corresponding curves for circuit 5 are presented in Fig. 2 f Here eps is specified as a relative residual norm:

$$eps = \frac{\|r_l\|}{\|r_0\|} \qquad (2)$$

where $r_0 = b - Ax_0$ and $r_l = b - Ax_l$.

The cost curve for the GMRES method are located lower than that of the QMR method. The same results were obtained for the all circuits we considered. It is also important to notice that high computational efficiency corresponds to low average specified error tolerance.

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The superior performance of GMRES is because the required sequence of Arnoldi vectors is relatively short in the considered problems. So, the above mentioned limitations of GMRES are not too essential while the GMRES technique exploits full information to construct the basis.

Error tolerance variation

Additional acceleration can be achieved in some cases by adapting the error tolerance of the linear solution to the current error of nonlinear iterations. The main idea is to reduce the time spent for the initial iterations that are far from the solution by specifying low accuracy for the linear solver and establishing high accuracy only for the final nonlinear iterations.

Several ways to coordinate errors at the linear and nonlinear levels can be pursued. The simplest one is to define a relative residual norm (2) as a stopping criterion for linear iterations. In our numerical experiments the following form of error tolerance was used as stopping criterion:

$$\delta = \gamma \frac{\|b_n\|^2}{\|b_{n-1}\|} \qquad (3)$$

$$\|Ax_l - b_n\| < \delta$$

Here $(\gamma < 1)$ is an experimental coefficient, n is the nonlinear iteration number.

Numerical experiments using (3) resulted in a simulation speed up of 25% and 50% for problems 1 and 3, respectively, when compared to using a constant average value of error tolerance.

Conclusion

a) The GMRES algorithm is preferred in comparison to QMR from the viewpoint of practical application in HB.

b) The average specified relative error tolerance for linear problems must not establish high accuracy; in practice a value in the range [0.01,0.5] provides the best efficiency for the considered test examples.

c) Implementation of an accuracy coordination principle between the linear and non-linear solutions generates new opportunities for reducing computational efforts for such large problems as HB analysis of RF circuits.

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